Abstract

This paper aims to develop a fractional order mathematical model addressing water pollution dynamics. The model is designed to elucidate the effect of pollutants and propose effective strategies for mitigating their spread in various water bodies such as rivers, lakes, oceans, or streams. Firstly, we formulate and analyze a nonlinear ordinary differential equations model that integrates a fractional derivative to capture the memory effect of pollutants in water. We initiate the analysis by establishing the existence of a unique positive and bounded solution. We then compute the basic reproduction number, which dictates the global dynamics of the model. Furthermore, we rigorously demonstrate the existence of a unique pollution-free equilibrium and the endemic equilibrium, and prove their global stability under appropriate assumptions on the basic reproduction number. Additionally, we conduct a global sensitivity analysis of the basic reproduction number to assess the variability in model predictions. Secondly, we enrich this initial model by extending it to a fractional partial differential system, incorporating spatial variables and diffusion terms to elucidate the transmission dynamics of pollutants in a spatially uniform environment. We establish the existence of a unique positive and bounded solution, along with the global stability of both pollution-free and endemic equilibria. To complement our theoretical findings, we perform numerical simulations using finite difference techniques and implemented via MATLAB.