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On bounds of holomorphic sectional curvature

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The aim of this paper is to study holomorphic sectional curvature and bounds on the holomorphic sectional curvature of an indefinite invariant submanifold of an indefinite complex space form.

Key words: invariant submanifold, complex space form.

INTRODUCTION

Let $\overline{M}^{n+p}_{s+t}$ (c), $c \neq 0$ be an indefinite complex space form of holomorphic sectional curvature c, then $\dim_{\mathbb{R}} \overline{M} = 2n+2p$ and index $= 2s+2t$, with $0 \leq s \leq n$ and $0 \leq t \leq p$. Let J be the almost complex structure and g the metric tensor of $\overline{M}^{n+p}_{s+t}$ (c) given by

$$g(X,Y) = -\sum_{i=1}^{s+t} X_i Y_i + \sum_{j=s+t+1}^{n+p} X_j Y_j \tag{1.1}$$

Let M^n_s be a 2n-dimensional indefinite invariant submanifold of index 2s immersed in $\overline{M}^{n+p}_{s+t}$ (c). A submanifold M of a Kaehler manifold is called invariant if each tangent space of M is mapped into itself by the almost complex structure of the Kaehler manifold (Chen & Ogiue, 1974). A Kaehler manifold of constant holomorphic sectional curvature is called a complex space form. We choose a local orthonormal frame field $\{e_1,\ldots,e_n;Je_1,\ldots,Je_n;e_{n+1},\ldots,e_{n+p};Je_{n+1},\ldots,Je_{n+p}\}$ on a neighbourhood of $\overline{M}^{n+p}_{s+t}$ in such a way that restricted to M^n_s, $e_1,\ldots,e_n;Je_1,\ldots,Je_n$ are tangent to M^n_s and $e_{n+1},\ldots,e_{n+p};Je_{n+1},\ldots,Je_{n+p}$ are normal to M^n_s. Moreover,

$\varepsilon_i = g(e_i,e_i) = g(Je_i,Je_i) = -1$, when $1 \leq i \leq s$

$\varepsilon_i = g(e_i,e_i) = g(Je_i,Je_i) = 1$, when $s+1 \leq i \leq n$

$\varepsilon_\alpha = g(e_\alpha,e_\alpha) = g(Je_\alpha,Je_\alpha) = -1$, when $n+1 \leq \alpha \leq n+t$

$\varepsilon_\alpha = g(e_\alpha,e_\alpha) = g(Je_\alpha,Je_\alpha) = 1$, when $n+t+1 \leq \alpha \leq n+p$.

Let $\overline{\nabla}$ be the covariant differentiation with respect to g and ∇ the covariant differentiation induced on M^n_s from g. Then the Gauss and Weingarten formulas are

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y) \text{ and } \overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N \tag{1.2}$$

for all $X,Y \in T(M^n_s)$ and $N \in T^{\perp}(M^n_s)$. Here h(X,Y) is the second fundamental form of the immersion, A_N the second fundamental tensor associated with N and $\nabla^{\perp}$ the connection on the normal bundle induced from $\overline{\nabla}$. Tensors h and A are related by the following equation: