SENRA Academic Publishers, Burnaby, British Columbia
Vol. 4, No. 2, pp. 1217-1219, June 2010
ISSN: 1715-9997

Canadian Journal of pure & applied sciences

ON SUBMANIFOLDS OF INDEFINITE COMPLEX SPACE FORM

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ABSTRACT

Sun (1994) showed that if M is a maximal spacelike submanifold of $\bar{M}_n^n(c)$ then either M is totally geodesic $(n \geq 2, c \geq 0)$ or $0 \leq S \leq -\frac{c}{4}n(n-1), (n \geq 2, c < 0)$. The purpose of this paper is to study the geometry of an n-dimensional compact totally real maximal spacelike submanifold M immersed in an indefinite complex space form $\bar{M}_p^{n+p}(c)$. In this manuscript we have shown that either the square of the length of second fundamental form S=0, implying M is totally geodesic for $c \geq 0, n > 1$ or $S \leq \frac{(1-n)(n+2p)}{4}c$ for $c < 0, n > 1$ and thus generalized Sun (1994) result.

Keywords: Totally real submanifold, complex space form, totally geodesic.

INTRODUCTION

Among all submanifolds of a Kaehler manifold there are two classes; the class of totally real submanifolds and the class of holomorphic submanifolds. A submanifold of a Kaehler manifold is called totally real (resp. holomorphic) if each tangent space of the submanifold is mapped into the normal space (resp. itself) by the almost complex structure of the Kaehler manifold, Chen and Ogiue (1974). A Kaehler manifold of constant holomorphic sectional curvature is called a complex space form, Wali (2005).

Let M be an n-dimensional totally real maximal spacelike submanifold isometrically immersed in a 2(n+p)-dimensional indefinite complex space form $\bar{M}_p^{n+p}(c)$ of holomorphic sectional curvature c and index 2p. We call M a spacelike submanifold if the induced metric on M from that of the ambient space is positive definite, Ishihara (1988). Let J be the almost complex structure of $\bar{M}_p^{n+p}(c)$. An n-dimensional Riemannian manifold M isometrically immersed in $\bar{M}_p^{n+p}(c)$ is called a totally real submanifold of $\bar{M}_p^{n+p}(c)$ if each tangent space of M is mapped into the normal space by the almost complex structure, Yano and Kon (1976).

Let h be the second fundamental form of M in $\bar{M}_p^{n+p}(c)$ and denote by S the square of the length of the second fundamental form h.

Sun (1994), proved that if M is a maximal spacelike submanifold of $\bar{M}_n^n(c)$ then either M is totally geodesic $(n \geq 2, c \geq 0)$ or $0 \leq S \leq -\frac{c}{4}n(n-1), (n \geq 2, c < 0)$. The purpose of this paper is to study an n-dimensional compact totally real maximal spacelike submanifold M immersed in an indefinite complex space form $\bar{M}_p^{n+p}(c)$.

Our main result is:

Theorem: Let M be an n-dimensional compact totally real maximal spacelike submanifold of $\bar{M}_p^{n+p}(c)$. Then either S=0, implying M is totally geodesic for $c \geq 0, n > 1$ or $S \leq \frac{(1-n)(n+2p)}{4}c$ for $c < 0, n > 1$.

LOCAL FORMULAS

Let $\bar{M}_p^{n+p}(c)$ be an indefinite complex space form of holomorphic sectional curvature c, dimension 2(n+p), $p \neq 0$ and index 2p. Let M be an n-dimensional totally real maximal spacelike submanifold isometrically immersed in $\bar{M}_p^{n+p}(c)$. We choose a local field of orthonormal frames

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