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Incorporating Sensitization in Mathematical Modeling of Alcoholism as an Infectious Disease

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Abstract: **Alcoholism is a problem that affects different aspects of our life in the society. In this study, alcoholism was taken and treated as an infectious disease. We developed a SISeTR model, which is a mathematical model of alcoholism as an infectious disease with five compartments i.e. susceptible, infected, sensitized, treated and recovered. The R_0 was developed and the model stability and the existence of equilibrium points determined. From the quantitative analysis of the study, it was found that the local stability existed when $R_0<1$. From the numerical analysis, we determined that Sensitization affects the dynamics of the model. We established that an increase in sensitization leads to a decline in the number of infections. This suggests that there is need for the relevant stakeholders to increase the rate of sensitization of the population as a measure of reducing the spread of alcoholism in the society.**

Keywords: **Alcoholism, Model, Sensitization, Infectious, Local Stability, Equilibrium, Global Stability.**

1. INTRODUCTION

Alcoholism can be described as an infectious disease that depict the incapability of an individuals to control their alcohol consumption due to their dependence an alcohol either emotionally or physically [3]. Alcoholism can then be presented as a problem that comes as a result of someone's preferences, community issues, environmental limitlessness, family background and preferences. Alcoholism is a major factor to the global influx of diseases, accidents and economic degradation [3]. Over the past years, most communities especially in Africa used alcohol as a form of socialization. Alcohol was used in celebrations to bring people together. It was a symbol of societal unity but all this has changed. Nowadays alcohol is used for commercial purposes by unscrupulous traders who will do everything to maximize the profit, including using harmful chemicals. Alcohol is readily available in shops and kiosks in most places, thus, many people can access it easily. This sometimes leads them to over drinking, which results in alcohol abuse

A report by World Health Organization (WHO) indicates that abuse of alcohol causes globe is an approximately 3.3 million death every year and 5.1 of diseases worldwide is attributed to alcoholism [7]. Thus, alcoholism is one of the leading global health risk behaviors due to its high risk and poor health and the negative impacts in our society.

Mathematical modeling is a means of illustrating a scientific awareness to ascertain on the effects that results from the changes in systems and to assist the stakeholders in decision making. Thus, this study put emphasis on the development of such scientific awareness on alcohol abuse. A mathematical model is made up of mini-models that can be grouped in to four categories i.e. studying, building, testing and using.

Studies have come up with several mathematical models to help in tackling alcoholism. Each model has been purposefully and specifically developed to address a specific objective on alcoholism and to come up with the most appropriate recommendations and findings. In these models, sensitization of the population was not factored in. Therefore, this study investigated the effect of sensitization of the population.

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In their research, Sharma and Samanta (2015) introduced treatment program in their alcohol abuse model. They showed that those who drink normally and ate treated always re-track back to drinking alcohol due to being in physical contact with fellow drinkers who are not in treatment [4]. They based their study on the fact that those in treatment compartment normally back to their drinking habit due to being in close contact with other alcoholics who have not been taken for treatment [4]. The treatment component was a good idea but again they didn't factor in a very important component which is the sensitization.

2. MODEL DESCRIPTION AND FORMULATION

The model is divided into five compartments at any time (t) which represent their status at the disease dynamic system. The compartments consist of Susceptible (S), the Infected (I), the Treated (T), the Sensitized (Se) and the Recovered (R).

The Susceptible compartment $S_{(t)}$ consists of human beings who are at risk of being infected with alcoholism upon being in con-tact with the alcoholics. The infected $I_{(t)}$ compartment consists of individuals who have become alcoholics and are involved in consumption of alcohol. Treated compartment $T_{(t)}$ consists of those who are infected and are undergoing treatment through detoxification by the use of methamphetamine and other drugs. The recovered compartment $R_{(t)}$ consists of those who were infected then either through treatment or through sensitization, they recovered from alcoholism. The ones who are in the Sensitization compartment $Se_{(t)}$ are the ones who are being sensitized to stop alcohol abuse.

The following are the definitions of the parameters used in the model

- Λ - The rate of recruitment.
- σ - The rate at which susceptible are infected.
- κ - The rate at which susceptible are sensitized
- θ - The rate at which the treated are re-infected.
- Ω - The rate at which the infected are treated.
- τ - The rate at which the infected recover
- λ - The rate at which the treated recover.
- ω - The rate at which the recovered can move back to susceptible.
- η - The rate at which the sensitized get infected.
- π - The rate at which the recovered get sensitized
- μ - Natural death
- δ - Death as a result of alcoholism.

The above model can be presented in a diagram as shown below in Fig 3.1

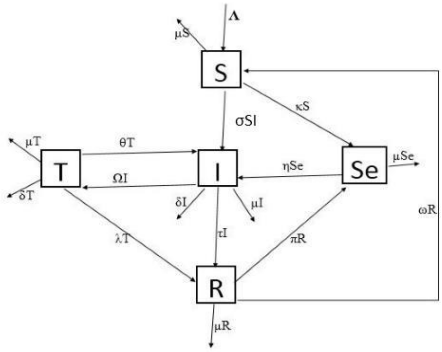

Figure 2.1: The Model flow

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Assumptions

The following assumptions were made on the model;

(i) All members of the population homogeneously mix i.e. each member of the population has a potential of becoming alcoholic.

(ii) The infected cannot go back to be susceptible.

(iv) (iii)The susceptible cannot be treated, they have to be infected first. The sensitized cannot die as a result of alcoholism.

The total population is given by

$N(t) = S(t) + I(t) + Se(t) + T(t) + R(t)$

We developed the following system of differential equations to help us find a solution to our problem.

$$\frac{dS}{dt} = \Lambda + \omega R - S(\mu + \sigma I + \kappa)$$

$$\frac{dI}{dt} = \sigma IS + \eta Se + \theta T - I(\Omega + \delta + \mu + \tau)$$

$$\frac{dSe}{dt} = \kappa S + \pi R - Se(\eta + \mu) \qquad (2.1)$$

$$\frac{dT}{dt} = \Omega I - T(\theta + \lambda + \delta + \mu)$$

$$\frac{dR}{dt} = \tau I + \lambda T - R(\pi + \mu + \omega)$$

3. DISEASE FREE EQUILLIBRIUM

At the Disease Free Equilibrium (DFE) there is no disease thus we set the values of the Infected, Treated and Recovered in the system of differential equations in (2.1) to zero. Thus, at equilibrium point $\frac{d}{dt} = 0$

$$I^* = 0$$

$$T^* = 0$$

$$R^* = 0$$

From the system of equations (2.1)

$$\frac{dS}{dt} = \Lambda + \omega R - S(\mu + \sigma I + \kappa)$$

$$S^* = \frac{\Lambda}{(\mu+\kappa)}$$

Also, from (2.1)

$$\frac{dSe}{dt} = \kappa S + \pi R - Se(\eta + \mu)$$

$$Se^* = \frac{\Lambda\kappa}{(\mu+\kappa)\mu}$$

Thus D.F.E = $(S^*, I^*, Se^*, T^*, R^*) = (\frac{\Lambda}{(\mu+\kappa)}, 0, \frac{\Lambda\kappa}{(\mu+\kappa)\mu} 0, 0)$

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3.1 STABILITY OF THE DISEASE-FREE EQUILIBRIUM

The stability of any given system refers to the way it behaves and the way in which its error is propagated. A system is stable if the eigenvalues of its Jacobian matrix lie within the stability region of that system.

3.2 LOCAL STABILITY OF DISEASE-FREE EQUILIBRIUM

According to Sirajo and Adabara (2013), the disease-free equilibrium is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$ [5].

From the D.F.E, the Jacobian matrix of the system is given by:

$$\begin{matrix} -(\kappa+\mu) & 0 & 0 & 0 & 0 \\ \kappa & -\mu & 0 & 0 & 0 \\ 0 & 0 & -(\Omega+\delta+\mu+\tau) & 0 & 0 \\ 0 & 0 & \Omega & -(\theta+\lambda+\delta+\mu) & 0 \\ 0 & 0 & \tau & \lambda & -(\pi+\mu+\omega) \end{matrix}$$

The eigenvalues (ξ_1, ξ_2, ξ_3, ξ_4 and ξ_5) of the Jacobian matrix are as shown below;

$$\xi_1 = -\mu$$

$$\xi_2 = -(\kappa+\mu)$$

$$\xi_3 = -\frac{1}{2}(\Omega+\delta+\lambda+\mu+\tau+\theta-\sqrt{\Omega 2 - 2\Omega\lambda + 2\Omega\tau + 2\Omega\theta + \lambda 2 - 2\lambda\tau + 2\lambda\theta + \tau 2 - 2\tau\theta + \theta 2})$$

$$\xi_4 = -\frac{1}{2}(\Omega+\delta+\lambda+\mu+\tau+\theta+\sqrt{\Omega 2 - 2\Omega\lambda + 2\Omega\tau + 2\Omega\theta + \lambda 2 - 2\lambda\tau + 2\lambda\theta + \tau 2 - 2\tau\theta + \theta 2})$$

$$\xi_5 = -(\pi+\mu+\omega)$$

ξ_1, ξ_2, ξ_4 and ξ_5 are negative. For ξ_3 to be negative the values under the square root must less than the sum of the other values i.e.

if we let $\frac{1}{2}(\Omega + \delta + \lambda + \mu + \tau + \theta)$ be D and $\sqrt{\Omega 2 - 2\Omega\lambda + 2\Omega\tau + 2\Omega\theta + \lambda 2 - 2\lambda\tau + 2\lambda\theta + \tau 2 - 2\tau\theta + \theta 2})$ be G

ξ_3 is negative if and only if D > G. The above condition will make all the eigenvalues to be negative. Since all eigenvalues are negative, it implies that the D.F.E is locally asymptotically stable [6].

4. BASIC REPRODUCTION NUNBER

In this section we use the Next Generation Matrix to determine the R_0. R_0 is the number new infections caused be one infected person in a population consisting of susceptible only [1]. When using the next generation matrix method to determine the R_0 we only consider the compartments with the infected individuals.

Therefore, we consider the infected and the treatment compartments i.e.

$$\frac{dI}{dt} = \sigma IS + \eta Se + \theta T - I(\Omega + \delta + \mu + \tau)$$

$$\frac{dT}{dt} = \Omega I - T(\theta + \lambda + \delta + \mu)$$

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Let F to be the number of new infections coming into the system and also V to be the number of new infections coming out of the given system;

$$F = \begin{bmatrix} \sigma SI + \eta Se \\ 0 \end{bmatrix} \text{ and } V = \begin{vmatrix} I(\Omega + \delta + \mu + \tau) \\ T(\theta + \lambda + \delta + \mu) \end{vmatrix}$$

Differentiating the above matrices with respect to I and T respectively we obtain;

$$F = \begin{bmatrix} \sigma S^* & 0 \\ 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} (\Omega + \delta + \mu + \tau) & 0 \\ 0 & (\theta + \lambda + \delta + \mu) \end{bmatrix}$$

The next generation matrix is given by

$$FV^{-1} = \begin{bmatrix} \frac{\sigma S^*(\theta+\lambda+\delta+\mu)}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)} & \frac{\sigma S^*\theta}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)} \\ 0 & 0 \end{bmatrix}$$

The spectral radius (ρ) of the next generation matrix (FV^{-1}) will always result in the basic reproductive number [6]. If we get the eigenvalues of FV^{-1} and pick the most dominant one, we get

$$\begin{bmatrix} \frac{\sigma S^*(\theta+\lambda+\delta+\mu)}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)} - A & \frac{\sigma S^*\theta}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)} \\ 0 & 0 - A \end{bmatrix} = 0,$$

where A denotes the eigenvalues, thus

$$A^2 - \frac{\sigma S^*(\theta + \lambda + \delta + \mu)}{(\theta + \delta + \mu + \tau)(\theta + \lambda + \delta + \mu)} A = 0$$

Therefore $A_1 = 0$ and $A_2 = \frac{\sigma S^*(\theta+\lambda+\delta+\mu)}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)}$

The dominant eigenvalue is A_2 therefore, $R_0 = \frac{\sigma S^*(\theta+\lambda+\delta+\mu)}{(\theta+\delta+\mu+\tau)(\theta+\lambda+\delta+\mu)}$, but $S^* = \frac{\Lambda}{(\mu+\kappa)}$

Therefore, the $R_0 = \frac{\sigma\Lambda}{(\theta+\delta+\mu+\tau)(\mu+\kappa)}$

5. ENDEMIC EQUILIBRIUM

At equilibrium points, $\frac{d}{dt} = 0$. Making I the subject in $\frac{dT}{dt} = \Omega I - T(\theta + \lambda + \delta + \mu)$ we obtain

$$I^* = \frac{T(\theta + \lambda + \delta + \mu)}{\Omega}$$

By calculation and substitution, we obtain the other values as

$$R^* = \frac{T(\tau(\theta + \lambda + \delta + \mu) + \Omega\lambda)}{\Omega(\pi + \mu + \omega)}$$

$$S^* = \frac{T(\tau\mu + \Omega\lambda + \tau(\theta + \lambda + \delta))\omega + \Lambda\Omega(\pi + \mu + \omega)}{(\sigma(\theta + \lambda + \delta + \mu)T + \Omega(\kappa + \mu))(\pi + \mu + \omega)}$$

$$Se^* = \left(\frac{\sigma\pi(\theta+\lambda+\mu)\tau(\theta+\lambda+\delta+\mu)+\Omega\lambda)T^2}{(\pi+\mu+\omega)(\sigma(\theta+\lambda+\delta)T+\Omega(\kappa+\mu))(\mu+\eta)\Omega}\right) + \left(\frac{((\kappa+\mu)\pi+\omega\kappa)(\tau(\theta+\lambda+\delta+\mu)+\Omega\lambda)\Omega T+\Lambda\Omega^2\kappa(\pi+\mu+\omega)}{(\pi+\mu+\omega)(\sigma(\theta+\lambda+\delta)T+\Omega(\kappa+\mu))(\mu+\eta)\Omega}\right)$$

$$T^* = \frac{I\Omega}{(\theta + \lambda + \delta + \mu)}$$

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We Let

$$a = (\theta + \lambda + \delta + \mu)$$

$$b = (\pi + \mu + \omega)$$

$$c = (\kappa + \mu)$$

$$M = (\omega\tau a^2\Omega\sigma + \tau a\Omega^2 cb\omega + \Omega^2\lambda\omega cb - \tau\mu\sigma ab - \Omega\Lambda\omega\sigma ab - \tau a^2\omega b)$$

$$M = (\Omega^2\omega bc(\tau a + \lambda) + \omega a\tau(\Omega\sigma a - ab) - (\omega\sigma ab(\tau\mu - \Omega\lambda) - \Omega^2\lambda\sigma a)$$

Therefore, by substituting a, b, c M and N, we obtain

$$T^* = \frac{-N \pm \sqrt{N^2}}{2M}$$

$$S^* = \frac{(\frac{-N \pm \sqrt{N^2}}{2M})\big(\tau\mu + \Omega\lambda + \tau(\theta + \lambda + \delta)\big)\omega + \Lambda\Omega(\pi + \mu + \omega)}{(\sigma(\theta + \lambda + \delta + \mu)(\frac{-N \pm \sqrt{N^2}}{2M}) + \Omega(\kappa + \mu))(\pi + \mu + \omega)}$$

$$I^* = \frac{(\frac{-N \pm \sqrt{N^2}}{2M})(\theta + \lambda + \delta + \mu)}{\Omega}$$

$$Se^* = \left(\frac{\boldsymbol{\sigma\pi}(\boldsymbol{\theta}+\boldsymbol{\lambda}+\boldsymbol{\mu})\tau(\theta+\lambda+\delta+\mu)+\Omega\lambda)(\frac{-N\pm\sqrt{N^2}}{2M})^2}{(\boldsymbol{\pi}+\boldsymbol{\mu}+\boldsymbol{\omega})\left(\boldsymbol{\sigma}(\theta+\lambda+\delta)(\frac{-N\pm\sqrt{N^2}}{2M})+\Omega(\kappa+\mu)\right)(\mu+\eta)\Omega}\right) + \left(\frac{((\boldsymbol{\kappa}+\boldsymbol{\mu})\boldsymbol{\pi}+\boldsymbol{\omega}\boldsymbol{\kappa})(\boldsymbol{\tau}(\theta+\lambda+\delta+\mu)+\Omega\lambda)\Omega T+\Lambda\Omega^2\kappa(\pi+\mu+\omega)}{(\boldsymbol{\pi}+\boldsymbol{\mu}+\boldsymbol{\omega})\left(\boldsymbol{\sigma}\left(\theta+\lambda+\delta(\frac{-N\pm\sqrt{N^2}}{2M})\right)+\Omega(\kappa+\mu)\right)(\mu+\eta)\Omega}\right)$$

$$R^* = \frac{(\frac{-N \pm \sqrt{N^2}}{2M})(\boldsymbol{\tau}(\theta + \lambda + \delta + \mu) + \ \Omega\lambda)}{\Omega(\pi + \mu + \omega)}$$

Endemic Equilibrium is given by;

(S*, I*, Se*, T*, R*) =

$$\frac{(\frac{-N \pm \sqrt{N^2}}{2M})\big(\tau\mu + \Omega\lambda + \tau(\theta + \lambda + \delta)\big)\omega + \Lambda\Omega(\pi + \mu + \omega)}{(\sigma(\theta + \lambda + \delta + \mu)(\frac{-N \pm \sqrt{N^2}}{2M}) + \Omega(\kappa + \mu))(\pi + \mu + \omega)},$$

$$\frac{(\frac{-N \pm \sqrt{N^2}}{2M})(\theta + \lambda + \delta + \mu)}{\Omega},$$

$$\left(\frac{\boldsymbol{\sigma\pi}(\boldsymbol{\theta}+\boldsymbol{\lambda}+\boldsymbol{\mu})\tau(\theta+\lambda+\delta+\mu)+\Omega\lambda)(\frac{-N\pm\sqrt{N^2}}{2M})^2}{(\boldsymbol{\pi}+\boldsymbol{\mu}+\boldsymbol{\omega})\left(\boldsymbol{\sigma}(\theta+\lambda+\delta)(\frac{-N\pm\sqrt{N^2}}{2M})+\Omega(\kappa+\mu)\right)(\mu+\eta)\Omega}\right) + \left(\frac{((\boldsymbol{\kappa}+\boldsymbol{\mu})\boldsymbol{\pi}+\boldsymbol{\omega}\boldsymbol{\kappa})(\boldsymbol{\tau}(\theta+\lambda+\delta+\mu)+\Omega\lambda)\Omega T+\Lambda\Omega^2\kappa(\pi+\mu+\omega)}{(\boldsymbol{\pi}+\boldsymbol{\mu}+\boldsymbol{\omega})\left(\boldsymbol{\sigma}\left(\theta+\lambda+\delta(\frac{-N\pm\sqrt{N^2}}{2M})\right)+\Omega(\kappa+\mu)\right)(\mu+\eta)\Omega}\right),$$

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$$\frac{\omega\Omega^2\, bc(\tau\mu + \Omega\lambda + \tau a - \lambda) + \Lambda\Omega\sigma a(b^2 - 1) + \sqrt{\omega b\Omega^2(\lambda - \tau\mu - \Omega\lambda - \tau a) + \Lambda\sigma a(1 - b^2)}}{2(\Omega^2\omega bc(\tau a + \lambda) - (\omega\sigma ab(\tau\mu - \Omega\lambda) + \omega a\tau(a\Omega\sigma - ab)) - \Omega^2\lambda\sigma a)},$$

$$\frac{(\frac{-N \pm \sqrt{N^2}}{2M})(\boldsymbol{\tau}(\theta + \lambda + \delta + \mu) + \ \Omega\,\lambda)}{\Omega(\pi + \mu + \omega)}$$

5.1 GLOBAL STABILITY OF ENDEMIC EQUILIBRIUM

According to Orwa (2014), a system of differential equations is said to have a unique endemic equilibrium if $R_0 > 1$, and it is globally asymptotically stable [2]. We shall prove this using the Lyapunov function as shown below;

L(S*, Se*, I*, T*, R*) = (S-S* − S*$ln\frac{S}{S*}$) + (I-I* − I*$ln\frac{I}{I*}$) + (Se-Se* − Se*$ln\frac{Se}{Se*}$) + (T-T* − T*$ln\frac{T}{T*}$)

+ (R-R* − R*$ln\frac{R}{R*}$)

So that;

$\frac{dL}{dt} = L$(S*, Se*, I*, T*, R*) = (S-S* − S*$ln\frac{S}{S*}$) + (I-I* − I*$ln\frac{I}{I*}$) + (Se-Se* − Se*$ln\frac{Se}{Se*}$) + (T-T* − T*$ln\frac{T}{T*}$)

+ (R-R* − R*$ln\frac{R}{R*}$)

$$\frac{dL}{dt} = \frac{dL}{dS}\frac{dS}{dt} + \frac{dL}{dI}\frac{dI}{dt} + \frac{dL}{dSe}\frac{dSe}{dt} + \frac{dL}{dT}\frac{dT}{dt} + \frac{dL}{dR}\frac{dR}{dt}$$

Therefore,

$$\frac{dL}{dt} = (\frac{S - S^*}{S})\frac{dS}{dt} + (\frac{I - I^*}{I})\frac{dI}{dt} + (\frac{Se - Se^*}{Se})\frac{dSe}{dt} + (\frac{T - T^*}{T})\frac{dT}{dt} + (\frac{R - R^*}{R})\frac{dR}{dt}$$

$$\frac{dL}{dt} = (1 - \frac{S^*}{S})\frac{dS}{dt} + (1 - \frac{I^*}{I})\frac{dI}{dt} + (1 - \frac{Se^*}{Se})\frac{dSe}{dt} + (1 - \frac{T^*}{T})\frac{dT}{dt} + (1 - \frac{R^*}{R})\frac{dR}{dt}$$

This gives;

$$\frac{dL}{dt} = (1 - \frac{S^*}{S})\Lambda + \omega R - S(\mu + \sigma I + \kappa) + (1 - \frac{I^*}{I})\sigma IS + \eta Se + \theta T - I(\Omega + \delta + \mu + \tau) + (1 - \frac{Se^*}{Se})\kappa S + \pi R - Se(\eta + \mu) + (1 - \frac{T^*}{T})\Omega I - T(\theta + \lambda + \delta + \mu) + (1 - \frac{R^*}{R})\tau I + \lambda T - R(\pi + \mu + \omega)$$

By expanding and simplifying the derivative above, we obtain;

$$\Lambda + \omega R - S(\mu + \sigma I + \kappa) + \Lambda S^* + \omega RS^* - S(\mu + \sigma I + \kappa)S^* + \sigma IS + \eta Se + \theta T - I(\Omega + \delta + \mu + \tau) + \sigma ISI^* + \eta SeI^* + \theta TI^* - I(\Omega + \delta + \mu + \tau)I^* + \kappa S + \pi R - Se(\eta + \mu) + \kappa SSe^* + \pi RSe^* - Se(\eta + \mu)Se^* + \Omega I - T(\theta + \lambda + \delta + \mu) + \Omega IT^* - T(\theta + \lambda + \delta + \mu)T^* + \tau I + \lambda T - R(\pi + \mu + \omega) + \tau IR^* + \lambda TR^* - R(\pi + \mu + \omega)R^*$$

We now let A to be the positive terms and B the negative terms in the equation above.

$$A = \Lambda + \omega R + (\mu + \sigma SI + \kappa)S^* + \kappa S + \pi R + (\eta + \mu)Se^* + \sigma SI + \eta Se + \theta T + (\theta + \delta + \mu + \tau)T^* + (\Omega)T + (\theta + \lambda + \mu)T^* + \tau I + \lambda T + (\pi + \mu + \omega)R^*$$

$$B = (\mu + \sigma SI + \kappa)S + \Lambda S^* + \omega RS^* + (\eta + \mu)Se + \kappa SS^* + \pi RS^* + (\Omega + \delta + \mu + \tau)I + \sigma II^* + \eta SeI^* + \theta TI^* + (\theta + \lambda + \mu)T + \Omega IT^* + (\pi + \mu + \omega)R + \tau IR^* + \lambda TR^*$$ Let $\frac{dL}{dt} = A - B$,

If we introduce the condition A<B. Then $\frac{dL}{dt} = \leq 0$,

Therefore $\frac{dL}{dt} = 0$, if and only if;

S = S*, I = I*, Se = Se*, T = T* and R = R*

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Therefore, the largest invariant set in

{(S*, I*, Se*, T*, R*) $\epsilon\Gamma$: $\frac{dL}{dt} = 0$ }, is E*, where E* is the endemic equilibrium point. This is the proof that the endemic equilibrium is globally asymptotically stable.

6. NUMERICAL SOLUTIONS

This section examines quantitative analysis of the system of differential equations of our model with the help of Runge-Kutta fourth order method using the Maple 8.21 software. The time duration of the spread of alcoholism was projected to be thirty years. First of all, we estimate the parameters to be used in calculations based on the literature that exist.

Table 1: Parameter Estimations

Symbol	Description	Value	Source
Λ	Rate of recruitment (birth rate)	2800	[10]
σ	Rate at which susceptible are infected	0.000009	[6]
κ	Rate at which susceptible are sensitized	0.02	Estimate
θ	Rate at which the treated are re-infected	0.2	[2]
Ω	Rate at which the infected are treated	0.5	[6]
τ	Rate at which infected recover	0.002	[2]
λ	rate at which the treated recover	0.1	[6]
ω	Rate at which recovered become susceptible	0.01	[6]
η	Rate at which the Sensitized get Infected	0.025	Estimate
π	Rate at which the recovered get Sensitized	0.2	Estimate
μ	Natural death rate	0.0054	[10]
δ	Rate of death as a result of alcoholism	0.002	[6]

6.1 Human Population Dynamics

Figure 6.1 shows dynamics of the populations comprising of Susceptible, Infected, Sensitized, Treated and Recovered. An increase in the number of the sensitized population leads to a decrease in the number of those who are at risk of becoming alcoholic. Initially, the population of the infected increases but after sometimes, it starts to decrease as a result of treatment and sensitization. It is also worth noting that initially the number of those recovered are less than the infected but after sometimes, the number of those who are recovering surpasses the infected. This is because many are exposed to treatment and sensitization. Also, the total population is increasing with time, this is because the rate of recruitment is higher than the rate at which people die either from natural causes or as a result of alcoholism. The graph of population dynamics is shown in figure 6.1 below.

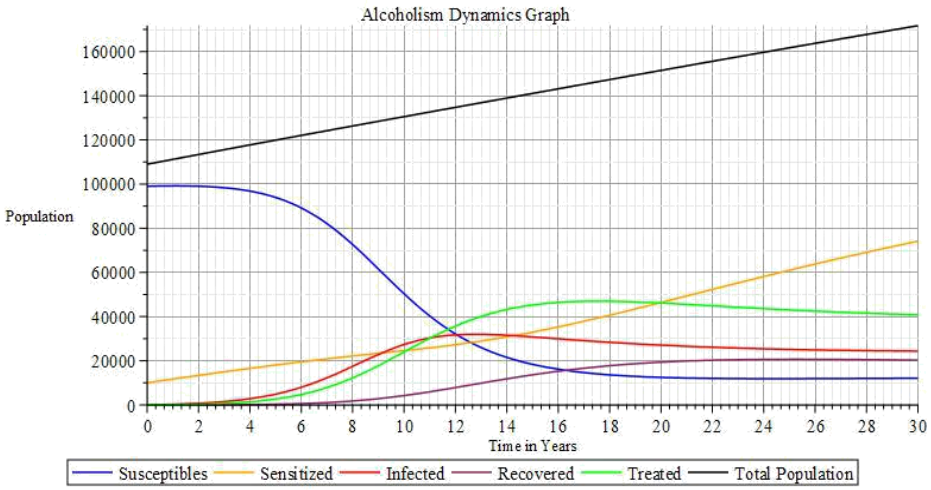

Figure 6.1: Alcoholism Dynamics Graph

6.2 Sensitized and Total Population

Figure 6.2 shows the graph of sensitized and total population. This is a logarithmic graph and its shows that with time, the gap between total population and the sensitized is reducing. This is because of a reduction in the number of susceptible due to sensitization and treatment. It shows that majority of the population are getting sensitized thus reducing the gap. With time, only a few people in the society remains un-sensitized.

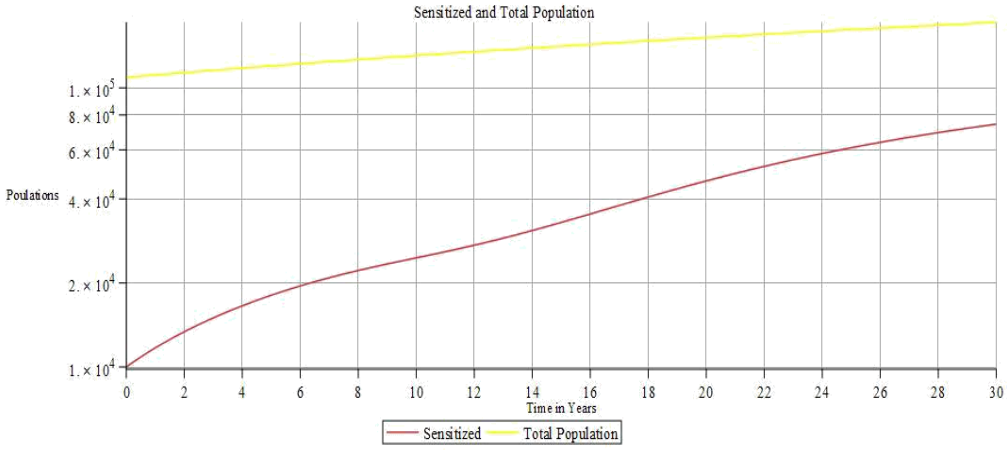

Figure 6.2: Sensitized and Total Population Graph

6.3 Sensitized and Infected

Figure 6.3 shows the graph of the sensitized and the infected population. From the graph it can be seen that at the initial stages both the sensitized and the infected are on the rise. After nine years, the number of infected equals the sensitized. Then, it surpasses it. Between nine and fourteen years, the number of infected are higher than those who are sensitized but the number of infected starts to reduce at twelve years or there about. After fourteen and half years the number of those who are sensitized becomes higher than the infected. From there, as the number of the sensitized increases the number of those infected decreases.

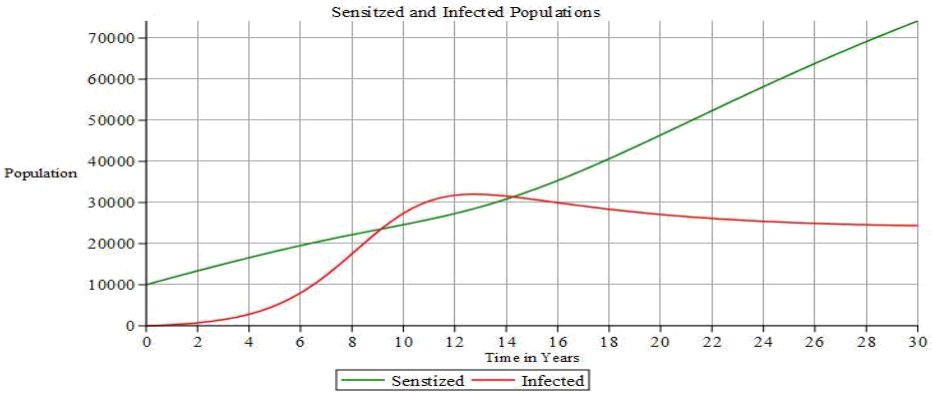

Figure 6.3: Sensitized and Infected Graph

6.4 Effects of Sensitization on Susceptible

Figure 6.4 shows the analysis of the rate of sensitization of the susceptible. The rate of sensitization of the susceptible is then varied to ascertain it's effect on the disease dynamics. It can be seen from the graph that an increase in the rate of sensitization of the susceptible leads to an increase in the number of sensitized populations. It also shows that when the rate of sensitization of the Susceptible is decreased, the number of the sensitized population also decreases. Therefore, we can confidently conclude that the rate of sensitization of the susceptible have a significant effect on the transmission of the disease. We can reduce alcoholism by increasing the rate at which the susceptible population is sensitized. The more we sensitize the masses, the less alcoholics we shall have in the population.

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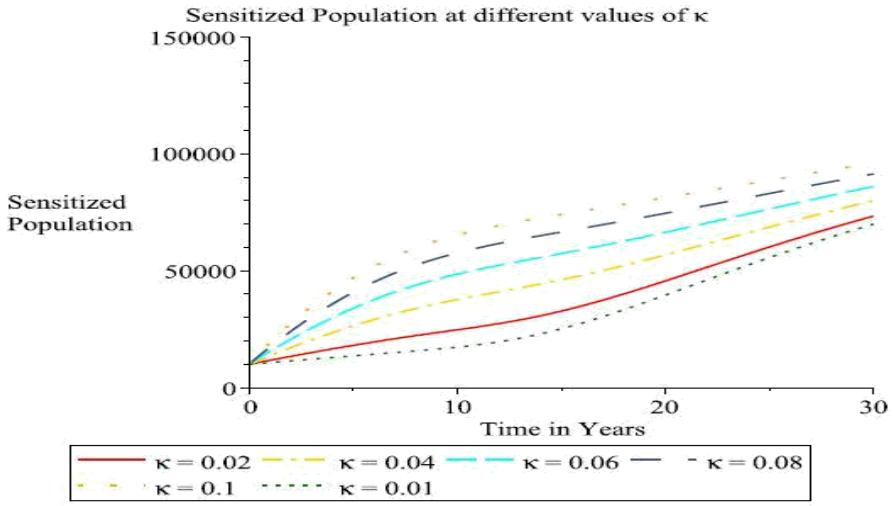

Figure 6.4: Sensitized Population at varied values of k

7. CONCLUSION

We have developed a Mathematical model of alcoholism as an infectious disease that incorporates sensitization and we have demonstrated that we can use compartmental model to get an estimation of how sensitization can affect the number of people who have the potential of becoming alcoholic. Through the developed model, we have studied and examined the dynamics and came up with a conclusion that sensitization affects the number of people that are infected, those who are susceptible, those who get treated and those who re-cover from alcoholism. We have also demonstrated that as we increase the number of those who are sensitized, the number of those who are susceptible decreases therefore significantly reducing the number of people who can become alcoholic. We established that the points of equilibrium exist. In our analysis we have shown that the D.F.E is globally stable when $R_0 < 1$. The effect of this is that to help control the rate of trans-mission of alcohol, we need to keep the R_0 to be below 1.

Just as the analytical solution gives us the equilibrium when $R_0 < 1$, the numerical solutions in figure 6.1 also gives us some level of stability. It is observed that with time when the population of the susceptible are exposed to sensitization, the number of susceptible reduces significantly. Over the years, majority of the initial population get sensitized and the rate of infection reduces. This means that sensitization has an impact on the dynamics of the model.

8. DATA AVAILABILITY

The data used in the analysis of the alcoholism model were obtained from previously published articles and which have been cited accordingly. Some of the parameter values are assumed and others are taken from published articles. These articles are cited at relevant places within the text as references.

CONFLICT OF INTEREST

The authors of this publication declare that there is no conflict of interest regarding the publication of this manuscript.

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